(54) Method and apparatus for the rendering of geometric volumes

(57) A method and apparatus is provided to directly render volumes from volume data whereby the resolution of the volume is not lost and the volume data is completely interactive with geometric data. The volume 150 or portion (slice) of the volume to be rendered is aligned to a geometric primitive 160 and a mapping function is generated relating the geometric primitive to the volume or volume portion. The mapping function relates each voxel in volume space (U, V, W) to an element or point of the primitive. Thus, the volume is displayed as a function of the geometric primitive the volume or volume portion is mapped to. In the preferred embodiment the volume or portion of volume to be rendered is bound by one or more geometric primitives and a mapping is generated relating the geometric primitive, such as a 3-D polygon, to the volume. Thereafter, any geometric operation performed on the volume is accomplished by performing the operation on the geometric primitive and using the mapping function to translate the volume into geometry space whereby the volume data is displayed. By defining the volume as a function of geometric primitives such as a polygon, geometric operations, such as clipping or rotating, may be performed easily on the primitive and the data is used to look up through the mapping function to determine the corresponding volume data to be displayed.

Fig. 4a
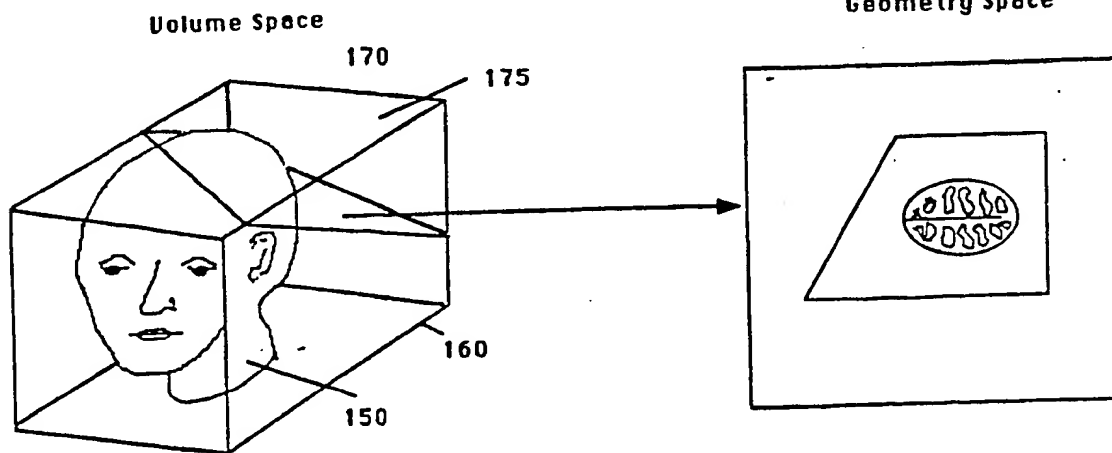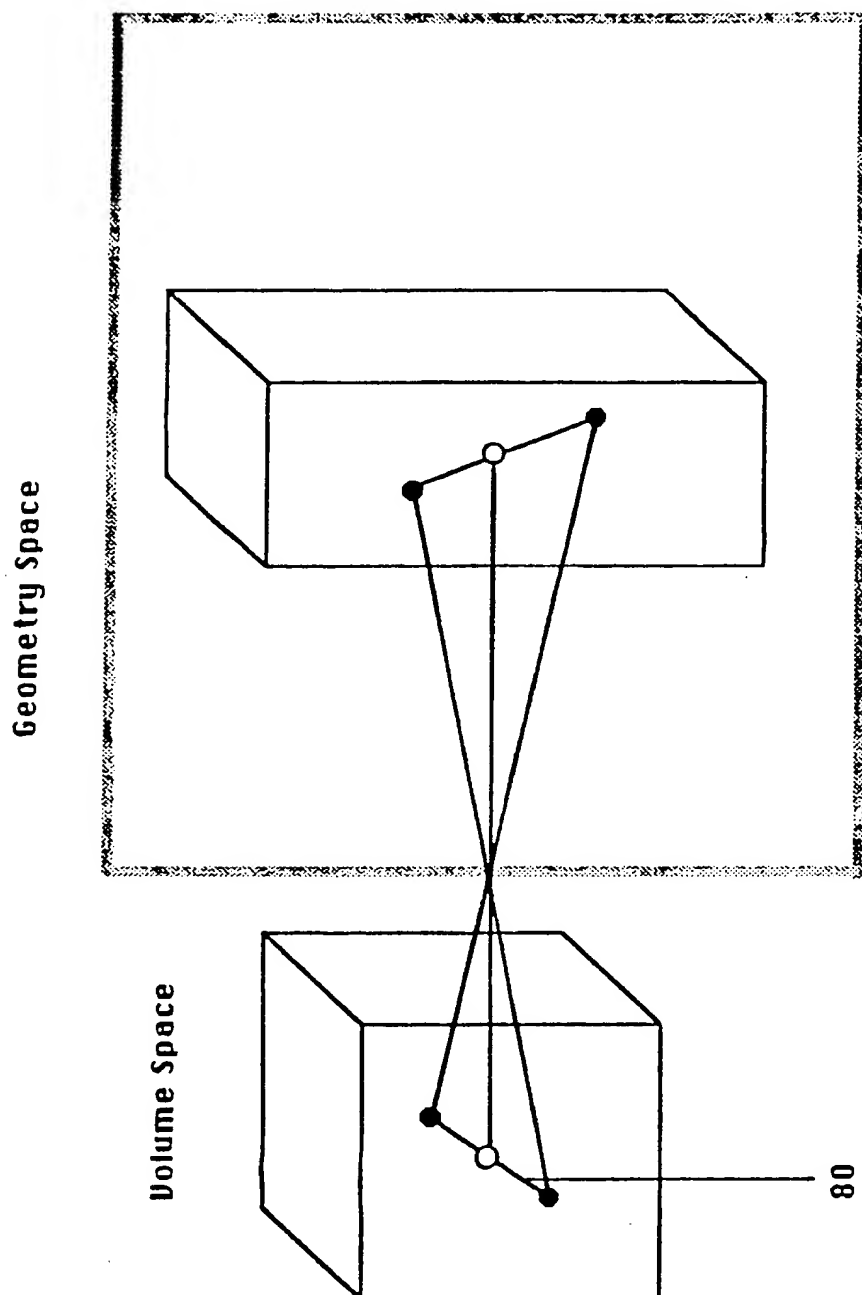
Fig. 4b

Fig. 1
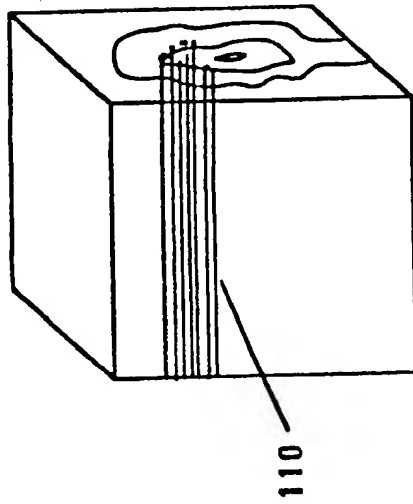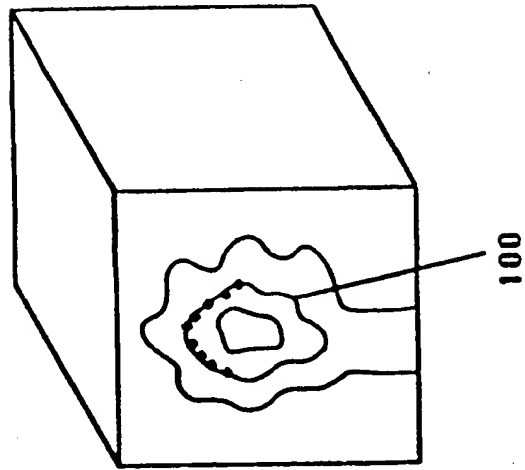
Display Monitor 7

Memory 4

CPU 3

I/O Circuit 2

1

Mass Memory 6

8

5

Geometry Space

Volume Space

80

Fig. 2

Fig. 3c

Fig. 3b

110

Fig. 3a

100

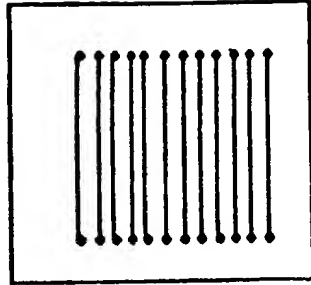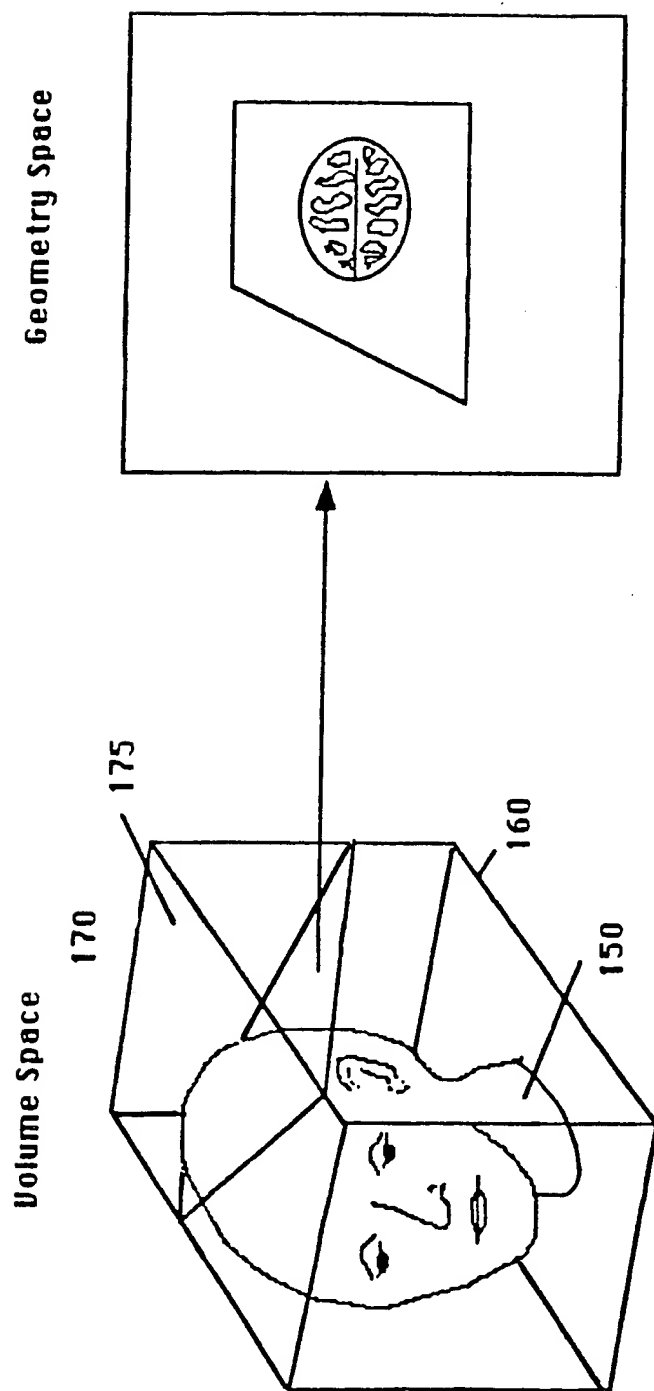Geometry Space

Fig. 4b
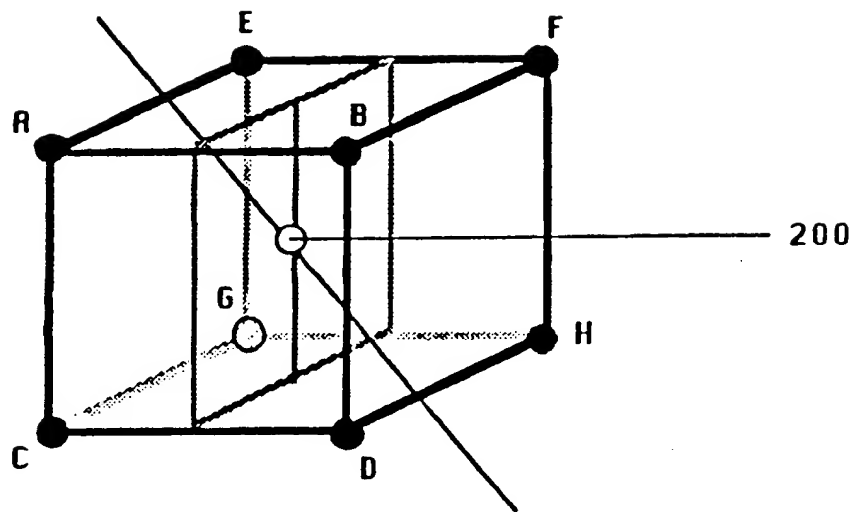
Volume Space

175

170

160

150

Fig. 4a

Fig. 5

-1-

METHOD AND APPARATUS FOR THE RENDERING OF GEOMETRIC VOLUMES

BACKGROUND OF THE INVENTION

1.   FIELD OF THE INVENTION:

This method and apparatus of the present invention relates to the

5   field of computer graphics more specifically, it relates to the rendering of volume
data on a computer display.

2.   ART BACKGROUND:

The technique of rendering volumes, that is data having three

10   dimensional aspects, is frequently used in the computer graphics field for
applications such as the display of images generated through x-ray
crystallography, computer tomography, positron emission tomography and
computational fluid dynamics.

15   Volume data is represented by a three dimensional array of
volume elements referred to as "voxels". Associated with each voxel are three
integer coordinates representing its location in volume space and at least one
integer value called its density, representing some property at the location, e.g.
temperature or composition, which is not directly related to rendering attributes.

20   On the other hand, geometric data is represented by an x, y, z coordinate
position and associated rendering attributes such as color. Geometric data
unlike volume data, may be easily rendered for display because the data
provides the rendering information necessary to display a geometric image;
volume data contains information related to the volume object itself.

25

Therefore, in order to display volume data, the volume data must
be translated out of volume space into a format compatible with the display
system space. There are basically two techniques to render volumes on a

computer graphics display: geometric and volumetric. In the geometric approach the volume data is fitted to geometric primitives which produces a model comprises of those primitives. In the volumetric approach the volume is rendered directly using either multiplaner reprojection or raycasting techniques.

5

Fuchs, Optimal Surface Reconstruction for Planer Contours, CACM 20 (1977), describes a geometric algorithm for rendering surfaces contained in volumetric data by reconstructing a 3-dimensional surface from a set of contours drawn on serial slices through the volume. The contours are commonly obtained using semi-automatic edge following routines, which are then tiled using geometric primitives such as triangles, polygons, or bi-cubic patches. These primitives form 3-D polygonal models which can be rendered using conventional techniques. A similar technique for surface reconstruction is described in Cline et al., U.S. Patent No. 4,729,098. However, the extraction of surface contours frequently requires operator intervention. Large numbers of polygons are required to retain detail. In addition, because the volume data is now represented only by the surface data of the volume; the raw volume data is lost and partial surfaces or objects cannot be displayed.

In an effort to retain surface detail at the voxel level the Cuberille method was developed (Gordon, Image Space Shading of 3-Dimensional Objects, Computer Vision Graphics and Image Processing 29, 377-393 (1985)). This technique creates a binary volume compressed for rendering efficiency representing a surface boundary. The faces of the voxels are rendered as shaded polygons. This technique does not allow for semi-transparent or partial surfaces. Another geometric algorithm, known as Marching Cubes, calculates the intersection of the surface with the voxel cube and generates triangles that give a highly detailed surface approximation. See, Lawrenson, et al. Marching

Cubes: A High Resolution 3-D Surface Construction Algorithm, Computer

Graphics, Siggraph 1987 Proceedings, pages 163-169 (July, 1987). Even

more detail is provided by the dividing cubes algorithm, which represents the

boundary as a set of points. See Cline, et al.,Two Algorithms For The

5   Reconstruction Of Surface From Tomograms, Medical Physics June 1988 and

Cline, U.S. Patent No. 4,719,585.

However, these methods provide for the surface extraction of the

volume; the raw volume data is not maintained. Therefore, any further

10   interaction with the raw volume data is not possible because only the volume

surface information is retained. Thus, such functions as partial volumes,

transparency, translucency and interaction with a volume, such as picking (i.e.,

reading data values from volumes or writing data values from volumes) and the

imposition of a geometric object on the volume, cannot be performed.

15

In the compositing technique, the volume data is preprocessed by

classifying each voxel according to various rendering attributes such as color,

opacity and texture thereby creating a multiplicity of volume representations.

The different volume representations are then combined to form a single

20   volume representation to be rendered on a display device. See Drebin, et al.,

Volume Rendering, Computer Graphics Siggraph proceedings, pages 65-74,

August 1988; LaVoy, Display Of Surfaces From Volume Data, IEEE Computer

Graphics and Applications (May 1988) and Goldwasser, et al., U.S. Patent No.

4,737,921. However, this technique does not provide for a continuous mapping

25   function. Thus, if the output image to be rendered is significantly larger than the

original volume data, significant aliasing occurs.

Raycasting is a simple way of direct voxel rendering. Roth, Raycasting For Modeling Solids, Computer Graphics and Image Processing 18, pages 109-144; Sabella, A Rendering Algorithm For Visualizing 3-D Scaler Fields, Computer Graphics Siggraph 1988 Proceedings, pages 51-55, August

5   1988; Upson et al., V-Buffer Visible Volume Rendering, Computer Graphics, Siggraph 1988 Proceedings, pages 59-64, August 1988. In the raycasting technique a ray is emitted at each pixel through the volume along the view-vector. The values determined along the ray are combined to be a single value and displayed on the display output. There have been many algorithms

10   developed to determine the output rendered by the ray. For example, the additive reprojection algorithm produces an x-ray like image by averaging the intensities of volume points along the ray. Another raycasting technique assigns color and opacity to density ranges allowing the volume to be viewed as a composition of translucent gels. (See Schlusselberg, et al., 3-Dimensional

15   Display Of Medical Image Volumes, Proceedings of the NCGA, March 1986). Kjiya, et al., Raytracing Volume Densities, Computer Graphics, Siggraph 1984 Proceedings, pages 165-173, describes true raytracing where additional rays are spawned at surface boundaries for the calculation of intersurface reflectance properties. In practice, however, true raytracing has not been used

20   because it is computationally intensive and expensive. In the multiplanar reprojection technique, one or more planes are moved through a volume displaying the density values mapped to some coloring function. However, this technique, like raycasting, is cumbersome and computation intensive and requires additional renderings for different perspectives.

25

SUMMARY OF THE INVENTION

It is therefore an object of the present invention to provide a method and apparatus for direct volume rendering using volume primitives.

5

It is an object of the present invention to provide a method and apparatus for rendering volumes incorporating volume-mapped geometric primitives.

10 It is further an object of the present invention to provide a method and apparatus for rendering volumes wherein the volumes may be interacted with geometric objects.

In the method and apparatus of the present invention a means is

15 provided to directly render volumes from volume data whereby the resolution of the volume is not lost and the volume data is completely interactive with geometric data.

The volume or portion of the volume to be rendered is aligned to a

20 geometric primitive and a mapping function is generated relating the geometric primitive to the volume or volume portion. The mapping function relates each voxel in volume space to an element or point of the primitive. Thus, the volume is displayed as a function of the geometric primitive the volume or volume portion is mapped to.

25

In the preferred embodiment the volume or portion of volume to be rendered is bound by one or more geometric primitive and a mapping is generated relating the geometric primitive, such as a 3-D polygon, to the

volume. Thereafter, any geometric operation performed on the volume is accomplished by performing the operation on the geometric primitive and using the mapping function to translate the volume into geometry space whereby the volume data is displayed. Preferably, the polygon which bounds the volume is

5 a rectilinear cube such as a normalized cube having its vertices range from -1 to +1.

By defining the volume as a function of geometric primitives such as a polygon, geometric operations, such as clipping or rotating, may be

10 performed easily on the primitive and the data is used to look up through the mapping function to determine the corresponding volume data to be displayed. Furthermore, geometric data may be easily interacted with the raw volume data because the raw volume data is being directly rendered in the geometry space.

BRIEF DESCRIPTION OF THE DRAWINGS

The objects features and advantages of the method and apparatus of the present invention will be apparent from the following description of the

5   invention in which:

FIG. 1 is a block diagram of a computer system for the rendering of volume data on a computer display.

10   FIG. 2 illustrates the mapping of volume elements in volume space to elements in geometry space.

FIG. 3a, 3b and 3c illustrate an application of the preferred embodiment of the present invention.

15

FIG. 4a and 4b illustrate the mapping and display of a volume bounded by a geometric cube.

FIG. 5 illustrates a trilinear mapping function.

NOTATION AND NOMENCLATURE

The detailed description which follows is presented largely in terms of algorithms and symbolic representations of operations on data bits

5   within a computer memory. These algorithmic descriptions and representations are the means used by those skilled in the data processing arts to most effectively convey the substance of their work to others skilled in the art.

An algorithm is here, and generally, conceived to be a self-

10   consistent sequence of steps leading to a desired result. These steps are those requiring physical manipulations of physical quantities. Usually, though not necessarily, these quantities take the form of electrical or magnetic signals capable of being stored, transferred, combined, compared, and otherwise manipulated. It proves convenient at times, principally for reasons of common

15   usage, to refer to these signals as bits, values, elements, symbols, characters, terms, numbers, or the like. It should be borne in mind, however, that all of these and similar terms are to be associated with the appropriate physical quantities and are merely convenient labels applied to these quantities.

20   Further, the manipulations performed are often referred to in terms, such as adding or comparing, which are commonly associated with mental operations performed by a human operator. No such capability of a human operator is necessary, or desirable in most cases, in any of the operations described herein which form part of the present invention; the operations are

25   machine operations. Useful machines for performing the operations of the present invention include general purpose digital computers or other similar devices. In all cases there should be borne in mind the distinction between the method operations in operating a computer and the method of computation

itself. The present invention relates to method steps for operating a computer in processing electrical or other (e.g., mechanical, chemical) physical signals to generate other desired physical signals.

5   The present invention also relates to apparatus for performing these operations. This apparatus may be specially constructed for the required purposes or it may comprise a general purpose computer as selectively activated or reconfigured by a computer program stored in the computer. The algorithms presented herein are not inherently related to a particular computer

10   or other apparatus. In particular, various general purpose machines may be used with programs written in accordance with the teachings herein, or it may prove more convenient to construct more specialized apparatus to perform the required method steps. The required structure for a variety of these machines will appear from the description given below.

15

DETAILED DESCRIPTION OF THE INVENTION

General System Configuration

5   Fig. 1 shows a typical computer-based system for rendering of volume data on a computer display according to the present invention. Shown there is a computer 1 which comprises three major components. The first of these is the input/output (I/O) circuit 2 which is used to communicate information in appropriately structured form to and from the other parts of the computer 1.

10   Also shown as a part of computer 1 is the central processing unit (CPU) 3 and memory 4. These latter two elements are those typically found in most general purpose computers and almost all special purpose computers. In fact, the several elements contained within computer 1 are intended to be representative of this broad category of data processors. Particular examples of suitable data

15   processors to fill the role of computer 1 include machines manufactured by Sun Microsystems, Inc., Mountain View, California. Other computers having like capabilities may of course be adapted in a straightforward manner to perform the functions described below.

20   Also shown in Fig. 1 is an input device 5, shown in typical embodiment as a keyboard. It should be understood, however, that the input device may actually be a card reader, magnetic or paper tape reader, or other well-known input device (including, of course, another computer or computerized data gathering lab equipment such as computer tomography

25   equipment). A mass memory device 6 is coupled to the I/O circuit 2 and provides additional storage capability for the computer 1. The mass memory may include other programs and the like and may take the form of a magnetic or paper tape reader or other well known device. It will be appreciated that the

data retained within mass memory 6, may, in appropriate cases, be incorporated in standard fashion into computer 1 as part of memory 4.

In addition, a display monitor 7 is illustrated which is used to
5   display messages or other communications to the user. Such a display monitor may take the form of any of several well-known varieties of CRT displays. Preferably, the display monitor 7 is a high resolution computer raster display device which can display the volume images rendered according to the process of the present invention. A cursor control 8 is used to select command modes
10   and edit the input data, such as, for example, the scale of the typeface, and provides a more convenient means to input information into the system.

Process Overview

15   In the method and apparatus of the present invention a volume object or portion of a volume object is aligned with one or more geometric primitives and a mapping function between the geometric primitive in geometry space and the volume object in volume space is generated. Through the use of the mapping function, transformation, viewing and other geometric operations
20   may be performed on the volume simply by performing the geometric operations on the primitives the volume is mapped to and translating the corresponding volume data.

The volume data, that is the voxels representative of the volume
25   object containing the attributes of the volume at a particular location in volume space, are stored in memory according to each voxel's location in a 3-D array of voxels representative of the volume object. To display the volume object the volume object is described in terms of at least one geometric primitive, for

example, a cube. The geometric primitive or primitives used should be of such a size and aligned in a manner that the volume or portion of the volume to be rendered can be mapped to the primitive. It is preferable that the primitive used is normalized in order to simplify subsequent computations that may be

5   performed such as magnification, translation or rotation of the image.

For example, referring to Fig. 2, a portion of a volume representative of a volume vector 80 can be mapped to a geometric vector and displayed in screen space. This vector once mapped, may be easily be rotated

10   and translated in the geometry space simply by performing the rotation/translation operations on the geometric vector the volume vector is mapped to and looking up the corresponding volume elements using the mapping function.

15   This is particularly useful for the technique of curved surface extraction using volume vector primitives For example, referring to Figs. 3a, 3b and 3c, a volume object may represent human tissue and/or organs wherein the volume contains capillaries 100 that are located throughout the volume. It is sometimes desirable to extract a capillary and to view it from a variety of

20   perspectives to determine the exact path or shape of the capillary through the tissues/organs representative of the volume. This is achieved by mapping volume vectors 110 to geometric vectors at points 120 along the curved surface, e.g., a capillary. Once the mapping is performed, the geometric vectors may be rotated and corresponding volume data is mapped thereby rendering a display

25   showing the capillary from different perspectives, for example the perspective illustrated in Fig. 3c.

Preferably, the volume is mapped to one or more geometric primitives which form a three dimensional object such as a rectilinear cube. The cube is aligned such that it bounds the volume. As illustrated in Fig. 4a, the volume object 150 is bounded by cube 160. A mapping function is generated

5   between the voxels comprising the volume and the {x,y,z} coordinates of the cube. Once the mapping function is generated, a variety of geometric operations may be performed on the volume object through operations performed on the cube. For example, a portion 175 of the volume 170 may be removed by performing clipping operations on the cube 160 and the resulting

10   image of the volume revealed can be easily displayed, as illustrated in Fig. 4b.

The mapping function provides a mathematical correspondence between each voxel of the volume object in volume space (also referred to as (u,v,w) space) and a vertex or grid point of the geometric primitive in {x,y,z}

15   space (geometry space). It can be said that the volume object is "fitted" to the geometric primitive such that boundaries of the volume are mapped to the boundaries of the geometric primitive.

Thus, the mapping function links the location(s) of the voxels

20   stored in memory to a particular x, y, z coordinate location of the geometric primitive(s) the volume data is mapped to. The mapping function used can be any order function. The function can be a zero order function wherein each voxel is mapped to the closest vertex in geometry space. For example, if a volume has the dimensions (10 x 20 x 30) and it is mapped to a normalized

25   rectilinear cube having its vertices located between -1 and 1, then the voxel at the position in the array (1, 1, 1) would be mapped to the vertex of the cube at the coordinate position {0, 0, 0} and the voxel at the position in the array (10, 20, 30) would be mapped to the vertex at coordinate location {1, 1, 1}. The voxels

between the position (1, 1, 1) and (10, 20, 30) in volume space are mapped to the closest vertex to the value computed using a size proportion calculation between volume space and geometry space. In the present example the u:x proportion is equal to 10:2 because the number of voxels in u is equal to 10 and

5   the number of vertices in x is 2 (-1 to 1). Thus to the mapping function may be $x_v = \frac{u_v}{5} - 1$. The function may also be a first order function referred to as a trilinear function wherein a weighted average calculation is performed on the eight closest vertices in geometry space to determine the location of the volume vertex in geometry space. Using a first order function produces a smoothing

10   effect thereby minimizing the amount of aliasing which occurs when the volume data is mapped. Referring to Fig. 5, a voxel 200 is mapped to a bounding cube by determining the eight closest verticies in geometry space illustrated by vertices, a, b, c, d, e, f, g and h. the coordinate location of the voxel is then determined according to the following equation:

15

$$X = (((a * FRACT(u) + (1\text{-}FRACT(u)) * b) *$$
$$FRACT(v) + (c * FRACT(u) + (1\text{-}FRACT(u)) * d) *$$
$$(1\text{-}FRACT(c)) * FRACT(w) + (e * FRACT(u)) +$$
$$(1\text{-}FRACT(u)) * p) * FRACT(v) + (g * FRACT(u)) +$$
20 $$(1\text{-}FRACT(u)) * n) * (1\text{-}FRACT(c)) * (1\text{-}FRACT(w))$$

where X is the geometric location of the voxel 200 at location {u,v,w} in volume space, a, b, c, d, e, f, g, and h are the eight {x,y,z} coordinate locations of the corners of the cube which surround the volume coordinate to be determined and FRACT is a function which determines the fractional portion of the

25   coordinate.

Once the mapping function has been determined, any geometric operation modification or manipulation which is performed on geometric objects may be performed on the volume object through the geometric primitive the

30   volume is mapped to. To achieve different viewing dimensions, aspect ratios or perspectives of the volume data, the geometric primitive is multiplied by a

transformation matrix which accordingly modifies the volume data mapped to the geometric primitive.

5   In addition, volume primitives are defined. The volume primitives are primitives such as a point, vector, polygon or ray which have definition in volume space and correspondingly in the geometry space according to the mapping performed. Thus, for example, a point has a location in {x,y,z} space and (u,v,w) space, a vector has two points in {x,y,z} and (u,v,w) space which define the end point of the vector. Similarly, a polygon has a plurality of points

10   in {x,y,z} and (u,v,w) space which define the boundaries of the polygon. A ray has a single point in {x,y,z} space which maps to a plurality of points in (u,v,w) space. The primitives may be used to add to, subtract or otherwise interact with the volume data. For example, a primitive may be used to slice away a portion of the volume to expose an interior surface. This is illustrated in Fig. 4a and 4b

15   wherein a polygon is used to slice away a portion of the volume to expose an interior surface of the volume. The primitives may also be used to generate a geometric image to interact with the volume in volume space where the effect may be displayed. This is particularly useful in medical emission therapy wherein the medical practitioner can preview the interaction of a partical beam

20   or laser beam created using primitives with human tissue which has been generated in the volume space. Thus a more realistic image is provided because the geometric image interacts with the raw volume data and not the surface extraction data of the volume.

25   While the invention has been described in conjunction with the preferred embodiment, it is evident that numerous alternatives, modifications, variations and uses will be apparent to those skilled in the art in light of the foregoing description.

CLAIMS

1.   A method for rendering volume objects on a computer graphics display, said method comprising the steps of:

5   storing volume data representative of at least a portion of the volume object in memory, said volume data represented in a (u,v,w) volume space;

defining at least one geometric graphics primitive to correspond to the volume object, said geometric graphics primitive represented in the {x,y,z} coordinate space and comprising at least one geometry coordinate

10   representative of a pixel on a computer graphics display;

aligning the volume data to the geometric graphics primitive; and

generating a mapping function which maps each volume data to geometry coordinate of the geometric graphics primitive;

whereby the volume data is displayed at the geometry coordinate

15   locations of the geometric graphics primitive the volume data are mapped to.

2.   The method of claim 1 wherein said mapping function is a zero order function whereby the volume data is mapped to the geometry coordinate closest to the ideal geometry coordinate, said ideal geometry coordinate

20   determined as a function of the proportion between the number of volume elements is each dimension to the length of the geometric graphics primitive in each dimension.

3.   The method of claim 1 wherein said mapping function is a first

25   order function whereby the volume data is trilinearly interpolated to determine the geometry coordinate data the volume data is mapped to.

4.   The method of claim 3 wherein the method for trilinearly interpolating the geometric data comprises the steps of:

determining the eight closest verticies in volume space which surround the volume data to be mapped;

5   computing the geometric coordinate the volume coordinate is mapped to as a function of the locations of the eight closest vertices and the distance away from the volume coordinate, said function defined as:

$$
\begin{aligned}
X = (((a * FRACT(u) &+ (1\text{-}FRACT(u)) * b) * \\
FRACT(v) + (c * FRACT(u) &+ (1\text{-}FRACT(u)) * d) * \\
(1\text{-}FRACT(c)) * FRACT(w) &+ (e * FRACT(u)) + \\
(1\text{-}FRACT(u)) * p) * FRACT(v) &+ (g * FRACT(u)) + \\
(1\text{-}FRACT(u)) * n) * (1\text{-}FRACT(c)) &* (1\text{-}FRACT(w))
\end{aligned}
$$

where x = the geometric location of the location (u,v,w) in volume space,

15   a, b, c, d, e, f, g, and h are the eight {x,y,z} coordinate locations of the corners of the cube which surround the volume coordinate to be determined and FRACT is a function which determines the fractional portion of the number.

5.   The method of claim 1 wherein said geometric graphics primitive

20   is selected from the group consisting of points, vectors, polygons and rays.

6.   The method of claim 1 further comprising the steps of:

performing geometric operations on the geometric primitive the volume data is mapped to thereby resulting in a modified geometric

25   primitive; and

displaying the volume data according to the modified graphics primitive using the mapping function determined whereby the volume data displayed is modified according to the geometric operations performed on the geometric primitive.

30

7.   The method of claim 6 wherein the geometric operation performed is the multiplication of the geometric primitive by a transformation matrix which rotates the geometric primitive and therefore the displayed volume data.

5   8.   The method of claim 6 wherein the geometric operation performed is the multiplication of the geometric primitive by a transformation matrix which changes the dimension of the geometric graphics primitive thereby changing the size of the displayed volume data.

10   9.   The method of claim 6 wherein the geometric operation performed is the multiplication of the geometric primitive by a transformation matrix which translates the geometric graphics primitive thereby translating the volume data displayed.

15   10.   The method of claim 6 wherein the geometric operation performed is the interaction of a geometric graphic object comprising geometric primitives with the geometric primitive the volume data is mapped to whereby the geometric graphic object is interacted with the volume data displayed.

20   11.   The method of claim 10 wherein the interaction of the geometric graphics object is to superimpose the geometric graphics object on the geometric graphics primitive the volume data is mapped to thereby superimposing the geometric graphics object on the volume data displayed.

12.   The method of claim 10 wherein the interaction of the geometric graphics object is the removal of a portion of the geometric graphics primitive the volume data is mapped to, said portion determined according to the size, shape and location of the geometric graphics object whereby the corresponding

5   portion of the volume data displayed is removed.

13.   The method of claim 10 wherein the interaction of the geometric graphics object is the extraction of portions of the geometric graphics primitive the volume data is mapped to, said portion determined according to the size

10   shape and location of the geometric graphics object, whereby the corresponding portion of the volume data extracted is displayed.

14.   A method for rendering volume objects on a computer graphics display wherein the volume object is displayed in screen space, said method

15   comprising the steps of:

storing in memory the volume data representative of at least a portion of the volume object, said volume data represented in a (u,v,w) volume space;

defining at least one geometric graphics primitive which bounds

20   the volume object in three dimensions, said geometric graphics primitive represented in the geometry space according to {x,y,z} geometry coordinates said {x,y,z} coordinates corresponding to pixels in the screen space of the computer graphics display;

generating a mapping function which maps each volume data to a

25   geometry coordinate of the geometric graphics primitive which bounds the volume object;

whereby the volume data is displayed at the geometry coordinate locations of the geometric graphics primitive the volume data are mapped to.

5   15.   The method of claim 14 wherein the geometric graphics primitive the volume data are mapped to is a rectilinear cube.

16.   The method of claim 14 wherein said mapping function is a zero order function whereby the volume data is mapped to the geometry coordinate

10   closest to the ideal geometry coordinate, said ideal geometry coordinate determined as a function of the proportion between the number of volume elements is each dimension to the length of the geometric graphics primitive in each dimension.

15   17.   The method of claim 14 wherein said mapping function is a first order function whereby the volume data is trilinearly interpolated to determine the geometry coordinate data the volume data is mapped to.

18. The method of claim 17 wherein the method for trilinearly interpolating the geometric data comprises the steps of:

determining the eight closest verticies in volume space which surround the volume data to be mapped;

5   computing the geometric coordinate the volume coordinate is mapped to as a function of the locations of the eight closest vertices and the distance away from the volume coordinate, said function defined as:

$$X = (((a \cdot FRACT(u) + (1-FRACT(u)) \cdot b) \cdot FRACT(v) + (c \cdot FRACT(u) + (1-FRACT(u)) \cdot d) \cdot$$
10     $$(1-FRACT(c)) \cdot FRACT(w) + (e \cdot FRACT(u)) + (1-FRACT(u)) \cdot p) \cdot FRACT(v) + (g \cdot FRACT(u)) + (1-FRACT(u)) \cdot n) \cdot (1-FRACT(c)) \cdot (1-FRACT(w))$$

where x = the geometric location of the location (u,v,w) in volume space,

15   a, b, c, d, e, f, g, and h are the eight {x,y,z} coordinate locations of the corners of the cube which surround the volume coordinate to be determined and FRACT is a function which determines the fractional portion of the number.

19. The method of claim 14 further comprising the steps of:

20   performing geometric operations on the geometric primitive the volume data is mapped to thereby resulting in a modified geometric primitive;

displaying the volume data according to the modified graphics primitive using the mapping function determined whereby the volume

25   data displayed is modified according to the geometric operations performed on the geometric primitive.

20. The method of claim 19 wherein the geometric operation performed is the multiplication of the geometric primitive by a transformation

30   matrix which rotates the geometric primitive and therefore the displayed volume data.

21.   The method of claim 19 wherein the geometric operation performed is the multiplication of the geometric primitive by a transformation matrix which changes the dimensions of the geometric graphics primitive

5   thereby changing the size of the displayed volume data.

22.   The method of claim 19 wherein the geometric operation performed is the multiplication of the geometric primitive by a transformation matrix which translates the geometric graphics primitive thereby translating the

10   volume data displayed.

23.   The method of claim 19 wherein the geometric operation performed is the interaction of a geometric graphic object comprising geometric primitives with the geometric primitive the volume data is mapped to whereby

15   the geometric graphic object is interacted with the volume data displayed.

24.   The method of claim 23 wherein the interaction of the geometric graphics object is to superimpose the geometrics graphics object on the geometric graphics primitive the volume data is mapped to thereby

20   superimposing the geometric graphics object on the volume data displayed.

25.   The method of claim 24 wherein the interaction of the geometric graphics object is the removal of a portion of the geometric graphics primitive the volume data is mapped to, said portion determined according to the size,

25   shape and location of the geometric graphics object whereby the corresponding portion of the volume data displayed is removed.

26.   The method of claim 24 wherein the interaction of the geometric
graphics object is the extraction of portions of the geometric graphics primitive
the volume data is mapped to, said portion determined according to the size,
shape and location of the geometric graphics object, whereby the

5   corresponding portion of the volume data extracted is displayed.

27.   An apparatus for rendering volume objects on a computer
graphics display, said apparatus comprising:

    means for storing the volume data representative of at least a

10   portion of the volume object in memory, said volume data represented in
a (u,v,w) volume space;

    means for defining at least one geometric graphics primitive to
correspond to the volume object, said geometric graphics primitive
represented in the {x,y,z} geometry coordinate space and comprising at

15   least one geometry coordinate representative of a pixel on a computer
graphics display;

    means for aligning the volume data to the geometric graphics
primitive; and

    means for generating a mapping function which maps each

20   volume data to a geometry coordinate of the geometric graphics
primitive;

    whereby the volume data is displayed at the geometry-coordinate
locations of the geometric graphics primitive the volume data are
mapped to.

25

28.   The apparatus of claim 27 wherein said means for generating a mapping function generates a zero order function whereby the volume data is mapped to the geometry coordinate closest to the ideal geometry coordinate, said ideal geometry coordinate determined as a function of the proportion

5   between the number of volume elements is each dimension to the length of the geometric graphics primitive in each dimension.

29.   The apparatus of claim 27 wherein said means for generating a mapping function generates a first order function and comprises a means for

10   trilinearly interpolating the volume data whereby the volume data is trilinearly interpolated to determine the mapping between geometry coordinate data and volume data.

30.   The apparatus of claim 27 wherein the means for trilinearly

15   interpolating the geometric data comprises:

means for determining the eight closest verticies in volume space which surround the volume data to be mapped;

means for computing the geometric coordinate the volume coordinate is mapped to as a function of the locations of the eight closest vertices and the

20   distance away from the volume coordinate, said function defined as:

$$
\begin{aligned}
X = & (((a \cdot FRACT(u) + (1\text{-}FRACT(u)) \cdot b) \cdot \\
& FRACT(v) + (c \cdot FRACT(u) + (1\text{-}FRACT(u)) \cdot d) \cdot \\
& (1\text{-}FRACT(c)) \cdot FRACT(w) + (e \cdot FRACT(u)) + \\
& (1\text{-}FRACT(u)) \cdot p) \cdot FRACT(v) + (g \cdot FRACT(u)) + \\
& (1\text{-}FRACT(u)) \cdot n) \cdot (1\text{-}FRACT(c)) \cdot (1\text{-}FRACT(w))
\end{aligned}
$$

25

where x = the geometric location of the location (u,v,w) in volume space,

a, b, c, d, e, f, g, and h are the eight {x,y,z} coordinate locations of the corners of the cube which surround the volume coordinate to be determined and FRACT is

30   a function which determines the fractional portion of the number.

31. The apparatus of claim 27 wherein said geometric graphics primitive is selected from the group consisting of points, vectors, polygons and rays.

5 32. The apparatus of claim 27 further comprising the steps of:

means for performing geometric operations on the geometric primitive the volume data is mapped to thereby resulting in a modified geometric primitive; and

means for displaying the volume data according to the modified

10 graphics primitive using the mapping function determined whereby the volume data displayed is modified according to the geometric operations performed on the geometric primitive.

33. The apparatus of claim 32 wherein the geometric operation

15 performed is the multiplication of the geometric primitive by a transformation matrix which rotates the geometric primitive and therefore the displayed volume data.

34. The apparatus of claim 32 wherein the geometric operation.

20 performed is the multiplication of the geometric primitive by a transformation matrix which changes the dimension of the geometric graphics primitive thereby changing the size of the displayed volume data.

35. The apparatus of claim 32 wherein the geometric operation

25 performed is the multiplication of the geometric primitive by a transformation matrix which translates the geometric graphics primitive thereby translating the volume data displayed.

36.   The apparatus of claim 32 wherein the geometric operation performed is the interaction of a geometric graphic object comprising geometric primitives with the geometric primitive the volume data is mapped to whereby the geometric graphic object is interacted with the volume data displayed.

5

37.   The apparatus of claim 32 wherein the interaction of the geometric graphics object is to superimpose the geometric graphics object on the geometric graphics primitive the volume data is mapped to thereby superimposing the geometric graphics object on the volume data displayed.

10

38.   The apparatus of claim 36 wherein the interaction of the geometric graphics object is the removal of a portion of the geometric graphics primitive the volume data is mapped to, said portion determined according to the size, shape and location of the geometric graphics object whereby the corresponding

15   portion of the volume data displayed is removed.

39.   The apparatus of claim 36 wherein the interaction of the geometric graphics object is the extraction of portions of the geometric graphics primitive the volume data is mapped to, said portion determined according to the size

20   shape and location of the geometric graphics object, whereby the corresponding portion of the volume data extracted is displayed.

40.   A apparatus for rendering volume objects on a computer graphics display wherein the volume object is displayed in screen space, comprising:

means for storing in memory the volume data representative of at least a portion of the volume object, said volume data represented in a (u,v,w) volume space;

means for defining at least one geometric graphics primitive which bounds the volume object in three dimensions, said geometric graphics primitive represented in the geometry space according to {x,y,z} geometry coordinates said {x,y,z} coordinates corresponding to pixels in the screen space of the computer graphics display;

means for generating a mapping function which maps each volume data to a geometry coordinate of the geometric graphics primitive which bounds the volume object;

whereby the volume data is displayed at the geometry coordinate locations of the geometric graphics primitive the volume data are mapped to.

41.·   The apparatus of claim 40 wherein the geometric graphics primitive the volume data are mapped to is a rectilinear cube.

42.   The method of claim 40 wherein said mapping function is a zero order function whereby the volume data is mapped to the geometry coordinate closest to the ideal geometry coordinate, said ideal geometry coordinate determined as a function of the proportion between the number of volume elements is each dimension to the length of the geometric graphics primitive in each dimension.

43.   The method of claim 40 wherein said mapping function is a first order function whereby the volume data is trilinearly interpolated to determine the geometry coordinate data the volume data is mapped to.

5   44.   The method of claim 43 wherein the method for trilinearly interpolating the geometric data comprises the steps of:

means for determining the eight closest verticies in volume space which surround the volume data to be mapped;

means for computing the geometric coordinate the volume coordinate is

10   mapped to as a function of the locations of the eight closest vertices and the distance away from the volume coordinate, said function defined as:

$$X = (((a * FRACT(u) + (1-FRACT(u)) * b) *$$
$$FRACT(v) + (c * FRACT(u) + (1-FRACT(u)) * d) *$$
15
$$(1-FRACT(c)) * FRACT(w) + (e * FRACT(u)) +$$
$$(1-FRACT(u)) * p) * FRACT(v) + (g * FRACT(u)) +$$
$$(1-FRACT(u)) * n) * (1-FRACT(c)) * (1-FRACT(w))$$

where x = the geometric location of the location (u,v,w) in volume space,

a, b, c, d, e, f, g, and h are the eight {x,y,z} coordinate locations of the corners of

20   the cube which surround the volume coordinate to be determined and FRACT is a function which determines the fractional portion of the number.

45.   The apparatus of claim 40 further comprising:

means for performing geometric operations on the geometric

25   primitive the volume data is mapped to thereby resulting in a modified geometric primitive;

means for displaying the volume data according to the modified graphics primitive using the mapping function determined whereby the volume data displayed is modified according to the geometric operations

30   performed on the geometric primitive.

46.   The apparatus of claim 45 wherein the geometric operation performed is the multiplication of the geometric primitive by a transformation matrix which rotates the geometric primitive and therefore the displayed volume data.

5

47.   The apparatus of claim 45 wherein the geometric operation performed is the multiplication of the geometric primitive by a transformation matrix which changes the dimensions of the geometric graphics primitive thereby changing the size of the displayed volume data.

10

48.   The apparatus of claim 45 wherein the geometric operation performed is the multiplication of the geometric primitive by a transformation matrix which translates the geometric graphics primitive thereby translating the volume data displayed.

15

49.   The apparatus of claim 45 wherein the geometric operation performed is the interaction of a geometric graphic object comprising geometric primitives with the geometric primitive the volume data is mapped to whereby the geometric graphic object is interacted with the volume data displayed.

20

50.   The apparatus of claim 49 wherein the interaction of the geometric graphics object is to superimpose the geometrics graphics object on the geometric graphics primitive the volume data is mapped to thereby superimposing the geometric graphics object on the volume data displayed.

25

51.   The apparatus of claim 50 wherein the interaction of the geometric
graphics object is the removal of a portion of the geometric graphics primitive
the volume data is mapped to, said portion determined according to the size,
shape and location of the geometric graphics object whereby the corresponding
5   portion of the volume data displayed is removed.

52.   The apparatus of claim 50 wherein the interaction of the geometric
graphics object is the extraction of portions of the geometric graphics primitive
the volume data is mapped to, said portion determined according to the size,
10   shape and location of the geometric graphics object, whereby the
corresponding portion of the volume data extracted is displayed.

53.   A method for rendering volume objects on a computer graphics
display substantially as hereinbefore described.

54.   An apparatus for rendering volume objects on a computer
graphics display substantially as hereinbefore described with reference
to the accompanying drawings.